Modeling temperature in traditional Nepalese dwellings heated by inverted downdraft gasifiers

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1 Abstract

Inefficient, unimproved cookstoves and open cooking fires are the cause of considerable respiratory distress and firewood depletion. In developing countries with significant heating needs, such as Nepal, smoke inhalation and firewood depletion result in poorer health outcomes for individuals, as well as decreased environmental health. We modeled the temperature in a traditional stone dwelling in Ghorepani, Nepal, heated by a scaled-up PekoPe African inverted downdraft gasifier. The firewood needs of such a traditional, uninsulated dwelling, even with improved heating technology, are enormous: fifteen metric tons per year! Insulation can reduce these firewood needs; applying insulation made of flax or plastic bottles could result in a 40 to 80 percent reduction in firewood usage (Nienhuys, 2001). Although improved stoves are one part of the heating puzzle, clearly, insulation is the other. We did not model added insulation in this traditional dwelling.

2 Introduction

Most individuals in developing countries use some form of biomass, either firewood, animal dung, or crop waste, for heating and cooking purposes. Burning this biomass in the traditional, unimproved heating and cooking stoves or open fires used inside dwellings leads to respiratory and other illnesses from high particulate matter(smoke) levels present. (Parajuli, Lee, and Shrestha 2016) Many individuals, non-governmental organizations, and government agencies since the 1950s have attempted to provide the Nepalese people with improved stoves for cooking and heating; these attempts have met with failure in many cases due to poor stove quality, inadequate stove distribution, and reluctance to adopt new technology. (Bhattarai and Risal, 2009)

Many of these improved cooking stoves, while an improvement over traditional fires, still fell short of their potential. In many cases, indoor particulate matter levels are reduced using chimney-vented stoves; these stoves do not provide any secondary combustion of particulate matter, carbon monoxide, or other pollutants: essentially, these stoves are a campfire in a metal or mud-brick box. These stoves are an improvement over traditional stoves, reducing particulate-matter concentrations by as much as 60 percent compared to traditional open fires.(Singh et al., 2012) However, there is a type of stove that will reduce particulate-matter emissions by as much as 90 percent, while retaining the capacity to burn fuels such as animal dung and crop residue as well as firewood. (Macarty, Still, and Ogle, 2010)

3 Methods

We model the temperature inside the building using Newton's Law of Cooling,

$$\frac{dT}{dt} = K(M(t) - T(t)) + H(t)$$

where $M(t)$ is the outside or ambient temperature, $T(t)$ is the temperature inside the building, and $H(t)$ is the rate at which ambient temperature would increase without heat loss.

We rewrite this linear nonhomogenous differential equation in the form

$$\frac{dT}{dt} + P(t)T(t) = Q(t)$$

where $P(t) = K, Q(t) = KM(t) + H(t)$ and the integrating factor is

$$\mu(t) = \exp \int K dt = e^{Kt}$$

so

$$\frac{d}{dt}[\mu(t)T(t)] = \mu(t)Q(t)$$

.

Solving for T, we get

$$\int \frac{d}{dt} e^{Kt} T(t) = \int e^{Kt} Q(t)$$

or

$$e^{kt} T(t) = \int e^{kt} Q(t) + C$$

Multiplying by $-e^{kt}$ we get

$$T(t) = e^{Kt} \left(\int e^{Kt} Q(t) + C \right)$$

Where the outside temperature $M(t)$ can be modeled as a sine wave with its minimum at $t = 0$ (6 AM) and its maximum at $t = 12$ (6 PM), $M(t) = M_0 - (B \cos \omega t)$. B is a positive constant, one-half of the daily temperature variation. M_0 is average outside temperature, and $\omega = \frac{\pi}{12}$ radians/hour.

$Q(t) = KM(t) + H(t)$. $M(t) = M_0 - (B \cos \omega t)$, so

$$Q(t) = KM_0 - (B \cos \omega t) + H_0$$

.

Setting $B_0 = M_0 + \frac{H_0}{K}$ we rewrite Q as

$$Q(t) = KB_0 - (B \cos \omega t)$$

. KB_0 is the daily average value of $Q(t)$.

$$T(t) = e^{-Kt} \left(\int e^{Kt} K(B_0 - B \cos \omega t) dt + C \right)$$

. Simplifying this etpression, we get

$$e^{-Kt}\int e^{Kt}KB_0 - e^{Kt}KB\cos\omega t + C$$

or

$$e^{-Kt}\Big(B_0\int e^{Kt}K - KB\int e^{Kt}\cos\omega t + C\Big)$$

which finally simplifies to

$$B_0 + Ce^{-Kt} - KBe^{-Kt}\int e^{Kt}\cos\omega t dt$$

We evaluate

$$\int e^{Kt}\cos(\omega t)dt$$

. Integrating by parts, where

$$\int fg' = fg - \int gf'$$

.

Let $f = \cos\omega t$, $g' = e^{Kt}dt$
$f' = -(\omega\sin\omega t)dt$, $g = \frac{e^{kt}}{k}$
and so

$$\int e^{Kt}\cos\omega t = \frac{e^{Kt}\cos\omega t}{K} + \frac{\omega}{K}\int e^{Kt}\sin(\omega t)dt$$

.

Applying integration by parts to $\int e^{Kt}\sin\omega t dt$ where
$f = \sin\omega t$, $g' = e^{Kt}dt$
$f' = (\omega\cos\omega t)dt$, $g' = e^{Kt}dt$
we get

$$\int e^{Kt}\cos\omega t dt = -\frac{-\omega^2}{K^2}\int e^{Kt}\cos\omega t dt + \frac{\omega e^{Kt}\sin\omega t}{K^2} + \frac{e^{Kt}\cos\omega t}{K}$$

Adding $\frac{\omega^2}{K^2}\int e^{Kt}\cos\omega t dt$ to both sides, we get

$$(\frac{\omega^2}{K^2}+1)\int e^{Kt}\cos\omega t dt = \frac{\omega e^{Kt}\sin\omega t}{K^2} + \frac{e^{Kt}\cos\omega t}{K} + C.$$

Dividing both sides by $\frac{\omega^2}{K^2}+1$ we get

$$\int e^{Kt}\cos\omega t dt = \frac{\frac{\omega e^{Kt}\sin\omega t}{K^2} + \frac{e^{Kt}\cos\omega t}{K}}{\frac{\omega^2}{K^2}+1}$$

Simplifying this function, we get

$$\int e^{Kt}\cos\omega t dt = \frac{e^{Kt}(K\cos\omega t + \omega\sin\omega t)}{K^2+\omega^2} + C$$

.

This is equal to

$$\frac{\cos\omega t + (\omega/K)\sin\omega t}{K(1+(\omega/K)^2)}e^{Kt}$$

.

Therefore,

$$T(t) = B_0 + Ce^{-Kt} - B\frac{\cos\omega t + (\omega/K)\sin\omega t}{K(1+(\omega/K)^2)} = B_0 + Ce^{-Kt} - BF(t)$$

where $F(t) = \frac{\cos\omega t + (\omega/K)\sin\omega t}{K(1+(\omega/K)^2)}$

We choose the constant C so that at 6 AM ($t = 0$) the value of the temperature T is equal to the initial temperature T_0.

$$C = T_0 - B_0 + BF(0) = T_0 - B_0 + \frac{B}{1+(\omega/K)^2}$$

.

Computing the value of the constant K is somewhat challenging. However, Borgkvist gave the volume of the stone walls and dirt floor of the Nepalese dwelling being modeled, as well as the inside and outside temperature of both the stone walls and the dirt floor. In her model, all temperatures, interior and exterior, are constant. The interior temperature for the stone walls of the Nepalese dwelling is 22 °C and the exterior temperature for the stone walls is 0 °C. The interior temperature for the dirt floors of the dwelling is also 22 °C, while the exterior temperature of the dirt floors is 9 °C.

The walls of the dwelling are 0.4 meters thick with an area of 59 square meters, while the roof has a surface area of 59 square meters and a thickness of 0.015 meters. The floor of the dwelling is 0.9 meters thick with an area of 50.3 square meters. Therefore, the volume of the stone walls and roof of the dwelling is $24m^3$, while the volume of the dirt floor is $45.3m^3$. . The heat capacity of stone is roughly 0.8 kJ/kg/°C, while we estimate the heat capacity of our dirt floor to be 0.9 kJ/kg/°C; the heat capacity of dry soil is approximately 0.8 kJ/kg/°C, while the heat capacity of wet soil is 1.3 kJ/kg/°C and the heat capacity of sandstone is 0.9 kJ/kg/°C. We assumed that our dirt floor consisted mostly of dry soil; Borgkvist only stated that the floor material was "stones and soil".

Rammed earth, the material that the dirt floor was made from, has a density of 1600 kilograms per cubic meter, while sandstone has a density of 2200 kilograms per cubic meter and granite has a density of 2800 kilograms per cubic meter. We estimate the density of the dirt floor at 1700 kilograms per cubic meter. The type of stone that made up the walls was not specified by Borgkvist; we estimate that the density of the stone walls is 2400 kilograms per cubic meter.

The total mass of the stone walls is 57,600 kilograms; the total mass of the dirt floor is 76,500 kilograms. The specific heat energy of the stone walls is (57,600)(0.8) or 48,000 kJ/°C. The specific heat energy of the dirt floor is (76,500)(0.9) or 68,850 kJ/°C. If the floor and walls of this structure were considered as a solid, homogeneous mass, 117,000 kilojoules of energy are required to raise the temperature of this mass by 1°C. To produce the 12°C temperature change in the dirt floor and the 21°C temperature change in the walls, (68,850)(12)+(48,000)(21) or 1,834,200 kJ of energy are needed. Dividing this by the 117,000 kJ needed to raise the temperature of this stone wall/dirt floor mass by 1°C, we get a mean temperature of 15.7°C for our mass.

The expenditure of energy in heating this dwelling is 270 kilowatt-hours per day, or 11.25 kWh/hour. Converting kilowatt-hours into kilojoules, we get an expenditure of 40,500 kilojoules per hour. Dividing this figure of 40,500 kilojoules by the 117,000 kilojoules it takes to heat our structure by one degree Celsius in the absence of heat loss, we get a rate of heat loss (at an average temperature of 15.7 °C) of 0.346°C per hour in the absence of external heat.

Inverted downdraft gasifiers, the type of stove being modeled here, often have an efficiency approaching 50 percent; the PekoPe stove with a 22 cm high and 18 cm diameter gasification chamber burns for 75 minutes using wood pellets. We assume that the burn time of inverted downdraft gasification stoves varies linearly with height, and the heat output varies linearly with surface area. Calculating the burn rate of the stove, we get $\frac{1.25\text{hours}}{0.22\text{meters}}$ or 5.68 hours of burn time per meter of combustion-chamber height. Calculating the heat output of the stove, we find that the radius of a stove 18 cm in diameter is 9 cm. Using the formula A = πr^2 we find the surface area of this stove is $254 cm^2$. (Roth, 2014)

We calculate the heat output of this stove by calculating the volume of this stove, $V = \pi r^2$. The volume of this stove is $5598 cm^3$ and the density of bulk wood pellets is $0.650 g/cm^3$. This stove burns 3638 grams of fuel in this 75 minute period, or 2910 grams per hour. The energy content of wood per kilogram is 17 MJ/kg, or 4.72 KWh per kilogram. With an efficiency of 50 percent, this stove produces 2.36 KWh/hour of energy. The amount of energy produced per square centimeter of surface area is $\frac{2.36}{254}$ or 0.00963 KWh/hour per cm^2 of stove surface area.(Roth, 2014)

These stoves, after burning their initial wood fuel load, burn the charcoal produced from the wood fire; this produces roughly 25 percent of the heat output of a wood fire for approximately 40 percent of the wood-fueled burn time. Alternatively, this charcoal can be removed, extinguished, and sold. We will assume that the occupants of this PekoPe-heated dwelling are removing and extinguishing the charcoal after burning their stove.

Returning to Newton's Law of Cooling, $\frac{dT}{dt} = K(M(t) - T(t)) + H(t)$, we find that $\frac{dT}{dt}$ = 0.346°C per hour, $M(t) = 0$, and $T(t) = 15.7$ °C. Solving for K, we get 0.346/15.7 or 0.022 for our constant K. The time constant 1/K for this dwelling is 45 hours, well within the range of time constants for dwellings in cold climates. The time constants for Swedish single-family dwellings range from 34 hours for lightly-constructed dwellings to 185 hours for heavily-constructed dwellings (Hedbrant, 2001)

We model a stove that has the capability of heating our dwelling by 1.5°C per hour in the absence of heat loss, and has a burn time of between three and six hours. Such a stove would produce $\frac{(1.5)(11.25)}{0.346}$ or 48.8 kWh per hour of energy. Its surface area would be $\frac{48.8 kWh/hour}{0.00963 kWh/hour/cm^3}$ or 5067 cm^3. The circular combustion chamber would have a diameter of Using the formula for the area of a circle, $A = \pi r^2$ and solving for r we get $\sqrt{\frac{A}{\pi}}$. $A = 5067 cm^3$ and so $r = 40$ cm. The diameter of the inner combustion chamber of this stove would be 80 cm. The stove burns at a rate of 5.68 hours per meter of burn time, so the height of the combustion chamber would be 6/5.68 or 1.05 meters. Its fuel consumption would be enormous; the energy content of wood is approximately 4.75 kWh per kilogram. At an efficiency of 50 percent, this stove would consume $\frac{48.8 kWh/hour}{2.38 kWh/kg}$ or approximately 20 kilograms of wood per hour! Clearly, with an uninsulated dwelling of this type, fuel costs to heat it would be enormous - even if such a fuel-hogging stove were operated 20 percent of the time, it would consume about 100 kg of wood per day; if a typical heating season lasted roughly 150 days at this fuel consumption level, fifteen metric tons of firewood would be needed per year!

We model this fuel-hogging stove using the model discussed earlier,

$$T(t) = B_0 + Ce^{-Kt} - B\frac{\cos\omega t + (\omega/K)\sin\omega t}{(1 + (\omega/K)^2)}$$

where

$$C = T_0 - B_0 + \frac{B}{1 + (\omega/K)^2}$$

. The daily temperature variation is approximately 6 degrees, so $B = 6$. $M_0 = 0$. $\omega = \frac{\pi}{12}$ or approximately 0.262. $B_0 = M_0 + \frac{H_0}{K} = \frac{1.5}{0.022}$. $B_0 = 68.2$. $(\omega/K) = (0.262/0.022) = 11.9$ The initial temperature of our dwelling is 15°C; at the start of the period, the occupants may not have fired their fuel-hogging stove in a day or two. $C = 15 - 68.2 + \frac{6}{1+(11.9)^2}$; $C = -53.15$

We model an initial six-hour firing, followed by a six-hour cool-down period, followed by a three-hour firing, followed by a nine-hour period when the occupants sleep, from 9 PM to 6 AM the following day.

The temperature of our dwelling can therefore be modeled by the function

$$T(t) = 68.15 - 53.15e^{-0.022t} - 6\frac{\cos 0.262t + 11.9\sin 0.262t}{142.8}$$

between $t = 0$ and $t = 6$. From $t = 6$ to $t = 12$, the dwelling is not heated at all; the stove has been completely extinguished. $B_0 = 0$ and $C = T(6)$; the contribution of $\frac{B}{1+(\omega/K)^2}$ can be considered to be negligible as this number is approximately 0.05. Six hours have elapsed since the start of our model period; we replace t with (t+6) in the trigonometric portion of our function in order to model the temperature variation six hours after $T(0)$. Therefore, our function is

$$T(t) = B_0 + Ce^{-Kt} - B\frac{\cos\omega(t+6) + (\omega/K)\sin\omega(t+6)}{(1 + (\omega/K)^2)}$$

. This model is inadequate, however: it models the temperature of our dwelling starting at $t = 6$, though the graph begins at $t = 0$. Our graph needs to be translated six units to the right. In order to translate a function $F(x)$ n units to the right, we subtract n units from the function's argument, yielding the function $F(x - n)$. Our new function is

$$T(t) = B_0 + Ce^{-K(t-6)} - B\frac{\cos\omega((t-6)+6) + (\omega/K)\sin\omega((t-6+6)}{(1 + (\omega/K)^2)}$$

which simplifies to

$$T(t) = B_0 + Ce^{-K(t-6)} - B\frac{\cos\omega t + (\omega/K)\sin\omega t}{(1 + (\omega/K)^2)}$$

.

In general, we can model the temperature in our building starting at time $t = n$ using the function

$$T(t) = B_0 + Ce^{-K(t-(n \mod 24)} - B\frac{\cos \omega t + (\omega/K)\sin \omega t}{(1+(\omega/K)^2)}$$

.

$$f(t) = 68.15 - 53.15e^{-0.022t} - 6\frac{\cos 0.262t + 11.9 \sin 0.262t}{142.8}$$

$f(6) = 21.1$

$$g(t) = 21.1e^{-0.022(t-6)} - 6\frac{\cos 0.262t + 11.9 \sin 0.262t}{142.8}$$

$g(12) = 19.1$

$$h(t) = 68.15 - 49.15e^{-0.022(t-12)} - 6\frac{\cos 0.262t + 11.9 \sin 0.262t}{142.8}$$

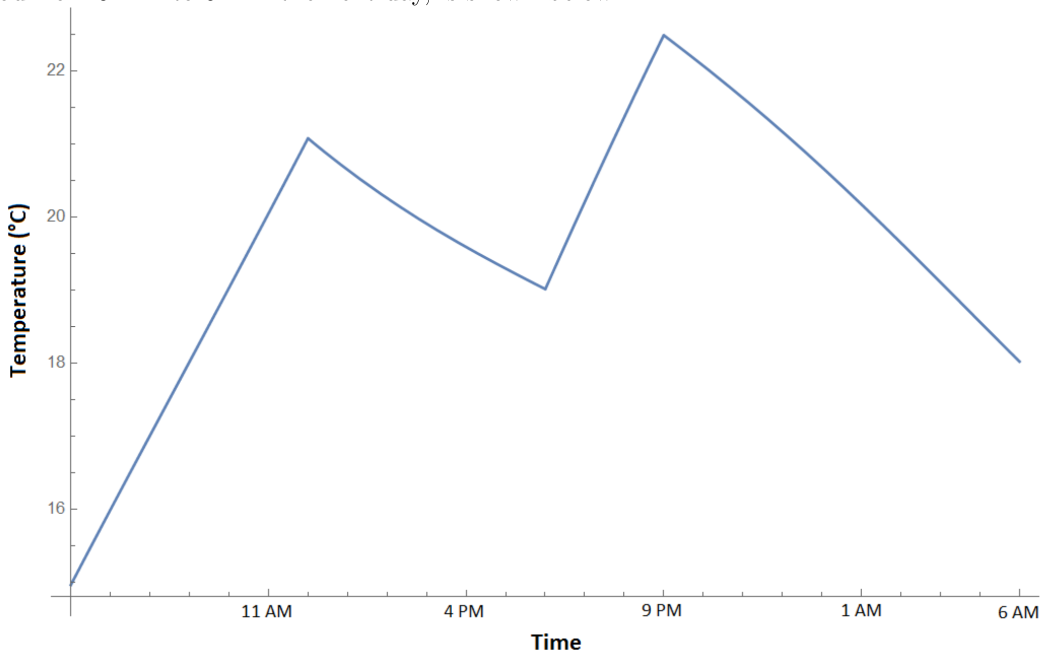
$h(15) = 19.1$

$$i(t) = 21.1e^{-0.022(t-15)} - 6\frac{\cos 0.262t + 11.9 \sin 0.262t}{142.8}$$

$$T(t) = \begin{cases} f(t) & 0 \leq t < 6 \\ g(t) & 6 \leq t < 12 \\ h(t) & 12 \leq t < 15 \\ i(t) & 15 \leq t < 24 \end{cases}$$

4 Graphs and Figures

The graph of this piecewise function, representing a six-hour firing from 6 AM to noon, a six-hour unheated period, a three-hour firing from 6 PM to 9 PM, then a nine-hour unheated period from 9 PM to 6 AM the next day, is shown below.



5 Conclusion

We were able to model the temperature in a traditional, uninsulated Nepalese dwelling subject to ambient temperature variation using Newton's Law of Cooling. The stove that we modeled was able to heat the building effectively, but only at the cost of an enormous amount of fuel; the stove would consume over 100 kg of wood per day, while traditional Nepali households consume only 20 to 40 kg of wood per day (Rural Integrated Development Service - Nepal). Clearly, better insulation is needed.

6 References

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