Bicycle and Fly Problem

Two cyclists 20 miles apart began riding directly toward each other. The instance they started, a fly on the handlebar of one bicycle started flying straight toward the other cyclist. As soon as it reached the other handlebar it turned and started back. The fly went back and forth this way, from handlebar to handlebar, until the two cyclists met.

If each cyclist had a constant speed of 10 miles per hour, and the fly flew at a constant speed of 15 miles per hour, how far did the fly travel?

1 Solution by infinite geometric series

The fly travels at 15 mph, while the cyclist travels at 10 mph. The distance between cyclist and fly decreases at 25 mph; this is their closing speed.

During the fly's first journey, from the first cyclist to the second cyclist, the fly travels for $\frac{20}{25}$ hours, or $\frac{4}{5}$ of an hour, traveling $\frac{15 \cdot 20}{25}$ or 12 miles.

During that time, the cyclists have each traveled $\frac{10 \cdot 20}{25}$ or 8 miles. The distance between the cyclists at the start of the fly's return journey is now 4 miles.

During the fly's second journey, the fly travels $\frac{4}{25}$ hour back to the first cyclist. It travels $\frac{4 \cdot 15}{25}$ or 2.4 miles in this time; the cyclists travel the remaining $\frac{10 \cdot 4}{25}$ or 1.6 miles.

During the fly's third journey, the distance between the cyclists has shrunken to 4-2(1.6) or 0.8 miles. The fly travels for $\frac{0.8}{25}$ hour at 15 mph, covering 0.48 miles. The cyclists cover the remaining 0.32 miles.

The distance between the cyclists on the fourth journey is a paltry 0.16 miles. The fly travels for $\frac{0.16}{25}$ hour at 15 mph, covering 0.096 miles.

At the end of four journeys, the fly has traveled $12 + \frac{12}{5} + \frac{12}{25} + \frac{12}{125}$ miles, or 14.976 miles.

The formula for the sum of a geometric series $a_1 + a_1 r + a_1 r^2 + a_1 r^3 + ... +$ where $-1 < r < 1$, is

$$\frac{a_1}{1 - r}$$

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Applying that formula for r = 0.2, we get $\frac{12}{1-0.2}$ or 15 miles.

2 Alternate method of solution

The cyclists travel 20 miles. After each cyclist travels 10 miles, the two cyclists will meet. This occurs one hour after they begin their journey. The fly travels 15 miles per hour, so the total distance traveled by the fly is 15 miles.